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Spin Squeezing of the Two Two-Level Atoms Interacting with a Binomial Field

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Abstract Spin squeezing of the two two-level atoms interacting with a binomial field has been investigated with the different initial conditions. It is shown that spin squeezing can be exhibited in the certain range of p and the degree of squeezing is dependent on p.

Keywords Spin squeezing · A binomial field · Two two-level atoms · Bell state

1 Introduction

The squeezing of quantum fluctuations is one of the most fundamental manifestations of the Heisenberg uncertainty relation, which is among the most important principles of quantum mechanics. Over the past decades, much attention has been paid to atomic spin squeezing [1, 2, 5, 7, 9, 10, 14, 16]. Why is much attention devoted to spin squeezing? On the one hand, spin squeezing has potential applications to ultra-precise spectroscopy and atomic clocks [12, 13]. On the other hand, some authors have established the relationship between the squeezing of the atoms and that of the radiation field [11]. Moreover, it has been shown that squeezed atoms can radiate a squeezed field [4].

In many previous studies of spin squeezing, the field was assumed to be a coherent field or a squeezed field. But less attention has been paid to the spin squeezing of atoms interacting with a binomial field. In the present work we will study the mechanism of the spin squeezing.

This paper is organized as follows. Section 2 studies the solution of the model. Section 3 gives the definition of spin squeezing. Section 4 investigates the numerical results of spin squeezing. A conclusion is presented in Sect. 5.

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2 The Model and Its Solution

The effective Hamiltonian of the model under consideration in this paper in the rotating-wave approximation can be written as ($\hbar = 1$)

$$H = H_0 + V, \tag{1}$$

where

$$H_0 = \omega a^+ a + \omega_0(\sigma_z^{(1)} + \sigma_z^{(2)}), \tag{2}$$

$$V = g(a^+\sigma_-^{(1)} + a\sigma_+^{(1)} + a^+\sigma_-^{(2)} + a\sigma_+^{(2)}), \tag{3}$$

where $a^+(a)$ is creation (annihilation) operator of the single-mode binomial field, $\sigma_z^{(1)}$, $\sigma_z^{(2)}$, $\sigma_-^{(1)}$, $\sigma_+^{(1)}$, $\sigma_-^{(2)}$, and $\sigma_+^{(2)}$ are the usual pseudospin operators acting in the space of atomic states and obey the commutation relations $[\sigma_+^{(1)}, \sigma_-^{(1)}] = \sigma_z^{(1)}$, $[\sigma_z^{(1)}, \sigma_\pm^{(1)}] = \pm\sigma_\pm^{(1)}$, $[\sigma_+^{(2)}, \sigma_-^{(2)}] = \sigma_z^{(2)}$, and $[\sigma_z^{(2)}, \sigma_\pm^{(2)}] = \pm\sigma_\pm^{(2)}$, ω and ω_0 are the frequencies of the binomial field and the atomic transition, respectively, g is the atoms-field coupling coefficient.

We consider that at $t = 0$ the two atoms are in one of the following Bell states

$$|\varphi_A(0)\rangle_+ = \frac{1}{\sqrt{2}}(|gg\rangle + i|ee\rangle), \tag{4}$$

$$|\varphi_A(0)\rangle_- = \frac{1}{\sqrt{2}}(|gg\rangle - i|ee\rangle), \tag{5}$$

$$|\phi_A(0)\rangle_+ = \frac{1}{\sqrt{2}}(|ge\rangle + i|eg\rangle), \tag{6}$$

$$|\phi_A(0)\rangle_- = \frac{1}{\sqrt{2}}(|ge\rangle - i|eg\rangle). \tag{7}$$

The complete Bell states have been generated in a thermal cavity [15]. Unified atomic state can be written as

$$|\psi_A(0)\rangle = \gamma_1|ee\rangle + \gamma_2|eg\rangle + \gamma_3|ge\rangle + \gamma_4|gg\rangle. \tag{8}$$

The above four Bell states can be obtained by choose appropriate values of γ_1, γ_2, γ_3, γ_4 in (8).

The light field is initially in the binomial state

$$|\psi_F(0)\rangle = |p, M\rangle = \sum_{n=0}^{M} B_n^M |n\rangle, \tag{9}$$

where

$$B_n^M = \left[\frac{M!}{n!(M-n)!} p^n (1-p)^{M-n}\right]^{1/2}, \tag{10}$$

where M, in this paper we take $M = 100$, is the maximum photon number present in the field and p is the characteristic probability of having each photon occurring. The single-mode binomial state of the quantized electromagnetic field was presented by Stoler et al.

in 1985 [6]. From (9) and (10) we can found that given any (finite) M, if $p=0$, $|p,M\rangle$ is reduced to the vacuum state $|0\rangle$. On the other hand, if $p=1$, we obtain the number state $|n=M\rangle$. Moreover, in the limit $p\to 0$ and $M\to\infty$, but with $pM=\alpha^2$ constant, the binomial distribution turns into a Poisson distribution and $|p,M\rangle$ becomes a coherent state $|\alpha\rangle$. It is interesting to found that by changing two parameters (p and M) in a binomial state, we can obtain fundamentally different states of the electromagnetic field.

The initial state of the system is a decoupled pure state, and the state vector can be written as

$$|\psi_{FA}(0)\rangle = |\psi_F(0)\rangle\otimes|\psi_A(0)\rangle = \sum_{n=0}^{M} B_n^M(\gamma_1|ee,n\rangle+\gamma_2|eg,n\rangle+\gamma_3|ge,n\rangle+\gamma_4|gg,n\rangle). \tag{11}$$

As the time goes, the evolution of the system in the interaction picture is governed by the state vector

$$|\psi_{FA}(t)\rangle = |a\rangle|ee\rangle+|b\rangle|eg\rangle+|c\rangle|ge\rangle+|d\rangle|gg\rangle, \tag{12}$$

where

$$|a\rangle=\sum_{n=0}^{M} a(n,t)|n\rangle, \tag{13}$$

$$|b\rangle=\sum_{n=0}^{M} b(n,t)|n\rangle, \tag{14}$$

$$|c\rangle=\sum_{n=0}^{M} c(n,t)|n\rangle, \tag{15}$$

$$|d\rangle=\sum_{n=0}^{M} d(n,t)|n\rangle. \tag{16}$$

Solving the Schrodinger equation

$$i\frac{\partial}{\partial t}|\psi_{FA}(t)\rangle = V|\psi_{FA}(t)\rangle, \tag{17}$$

we can obtain

$$\begin{aligned} a(n,t) &= \varepsilon_3(n+1) - i\sqrt{\frac{n+1}{2(2n+3)}}[\varepsilon_1(n+1)\sin(\sqrt{2(2n+3)}gt) \\ &\quad - \varepsilon_2(n+1)\cos(\sqrt{2(2n+3)}gt)] \quad (n=0,1,2,\dots,M), \end{aligned} \tag{18}$$

$$\begin{aligned} b(n,t) = c(n,t) &= \frac{1}{2}\varepsilon_1(n)\cos(\sqrt{2(2n+1)}gt) \\ &\quad + \frac{1}{2}\varepsilon_2(n)\sin(\sqrt{2(2n+1)}gt) \quad (n=1,2,3,\dots,M), \end{aligned} \tag{19}$$

$$\begin{aligned} d(n,t) &= \varepsilon_4(n-1) - i\sqrt{\frac{n}{2(2n-1)}}[\varepsilon_1(n-1)\sin(\sqrt{2(2n-1)}gt) \\ &\quad - \varepsilon_2(n-1)\cos(\sqrt{2(2n-1)}gt)] \quad (n=2,3,4,\dots,M), \end{aligned} \tag{20}$$

$$b(0,t)=c(0,t)=\frac{1}{2}(\gamma_2+\gamma_3)B_0^M\cos(\sqrt{2}gt)-i\frac{1}{\sqrt{2}}\gamma_4B_1^M\sin(\sqrt{2}gt), \tag{21}$$

$$d(0,t)=\gamma_4B_0^M, \tag{22}$$

$$d(1,t)=\gamma_4B_1^M\cos(\sqrt{2}gt)-i\frac{1}{\sqrt{2}}(\gamma_2+\gamma_3)B_0^M\sin(\sqrt{2}gt), \tag{23}$$

where

$$\varepsilon_1(n)=(\gamma_2+\gamma_3)B_n^M, \tag{24}$$

$$\varepsilon_2(n)=-i\sqrt{\frac{2}{2n+1}}(\gamma_1\sqrt{n}B_{n-1}^M+\gamma_4\sqrt{n+1}B_{n+1}^M), \tag{25}$$

$$\varepsilon_3(n)=\frac{1}{2n+1}(\gamma_1(n+1)B_{n-1}^M-\gamma_4\sqrt{n(n+1)}B_{n+1}^M), \tag{26}$$

$$\varepsilon_4(n)=\frac{1}{2n+1}(\gamma_4nB_{n+1}^M-\gamma_1\sqrt{n(n+1)}B_{n-1}^M). \tag{27}$$

3 The Definition of Spin Squeezing

It is necessary for us first to give a general definition [8] of what we mean by squeezing or reduction of quantum fluctuations. For two arbitrary operators A and B which obey the commutation relation $[A,B]=C$, the product of the uncertainties in determining their expectation values is given by

$$\Delta A\Delta B\geq\frac{1}{2}|\langle C\rangle|, \tag{28}$$

where $(\Delta A)^2=\langle A^2\rangle-\langle A\rangle^2$ and $(\Delta B)^2=\langle B^2\rangle-\langle B\rangle^2$. We will define squeezing if the uncertainty in one of the observables satisfies the relation

$$(\Delta A)^2<\frac{1}{2}|\langle C\rangle|\quad\text{or}\quad(\Delta B)^2<\frac{1}{2}|\langle C\rangle|. \tag{29}$$

Over the past decades, many different definitions of spin squeezing have been used depending on the context in which squeezing is considered [3, 5, 12, 16]. In this paper, spin squeezing parameters are based on angular-momentum commutation relations. From the commutation relation $[J_x,J_y]=iJ_z$, the uncertainty relation between different components of the angular momentum is given by

$$\Delta J_x\Delta J_y\geq\frac{1}{2}|\langle J_z\rangle|, \tag{30}$$

where

$$J_+=\sigma_+^{(1)}+\sigma_+^{(2)}, \tag{31}$$

$$J_-=\sigma_-^{(1)}+\sigma_-^{(2)}, \tag{32}$$

$$J_z=\frac{1}{2}(\sigma_z^{(1)}+\sigma_z^{(2)}), \tag{33}$$

$$J_x = \frac{1}{2}(J_+ + J_-), \tag{34}$$

$$J_y = \frac{1}{2i}(J_+ - J_-). \tag{35}$$

Without violating Heisenberg's uncertainty relation, it is possible to redistribute the uncertainty unevenly between J_x and J_y, so that a measurement of either J_x or J_y becomes more precise than the standard quantum limit $\sqrt{|\langle J_z\rangle|/2}$. States with this property are called spin squeezed states in analogy with the squeezed states of a harmonic oscillator. Consequently, two squeezing parameters can be written as

$$F_1 = (\Delta J_x)^2 - \frac{1}{2}|\langle J_z\rangle|, \tag{36}$$

$$F_2 = (\Delta J_y)^2 - \frac{1}{2}|\langle J_z\rangle|. \tag{37}$$

If the parameter F_1 (F_2) satisfies the condition $F_1 < 0$ ($F_2 < 0$), the fluctuation in the component J_x (J_y) is said to be squeezed.

4 Numerical Results and Discussion

In the light of the complexity of the analytical results of F_1 and F_2 given by (36) and (37), F_1 and F_2 are numerically solved. It is found that F_1 (F_2) for the atoms initially in Bell state $|\varphi_A(0)\rangle_+$ and Bell state $|\varphi_A(0)\rangle_-$ always has the same numerical results and the same regulation is discovered in Bell state $|\phi_A(0)\rangle_+$ and Bell state $|\phi_A(0)\rangle_-$. So the only two situations for the atoms initially in Bell state $|\varphi_A(0)\rangle_+$ and Bell state $|\phi_A(0)\rangle_+$ are shown as follows. Figure 1 shows spin squeezing for the atoms initially in Bell state $|\varphi_A(0)\rangle_+$. Figure 2 shows spin squeezing for the atoms initially in Bell state $|\phi_A(0)\rangle_+$.

On the other hand, by a lot of numerical computation, it is found that the fluctuation in the component J_y isn't squeezed for the atoms initially in Bell state $|\varphi_A(0)\rangle_+$ and the fluctuation in the component J_x isn't squeezed for the atoms initially in Bell state $|\phi_A(0)\rangle_+$.

4.1 Atoms Initially in Bell State $|\varphi_A(0)\rangle_+ = \frac{1}{\sqrt{2}}(|gg\rangle + i|ee\rangle)$

In the case of $\gamma_1 = \frac{i}{\sqrt{2}}$ and $\gamma_4 = \frac{1}{\sqrt{2}}$, the initial state of the atoms is reduced to Bell state $|\varphi_A(0)\rangle_+$. Figure 1 indicates the influence of the characteristic probability p on time evolution of F_1 for the maximum photon number $M = 100$, the atoms initially in Bell state $|\varphi_A(0)\rangle_+$.

According to our numerical computation, it is found that spin squeezing for J_x is exhibited in the certain area $0.98 < p < 1$. But the fluctuation of J_y isn't squeezed for any p. From these pictures, we can find the degree of spin squeezing is rapidly close to zero when p is close to 1.

4.2 Atoms Initially in Bell State $|\phi_A(0)\rangle_+ = \frac{1}{\sqrt{2}}(|ge\rangle + i|eg\rangle)$

In the case of $\gamma_2 = \frac{i}{\sqrt{2}}$ and $\gamma_3 = \frac{1}{\sqrt{2}}$, the initial state of the atoms is reduced to Bell state $|\phi_A(0)\rangle_+$. Figure 2 indicates the influence of the characteristic probability p on time

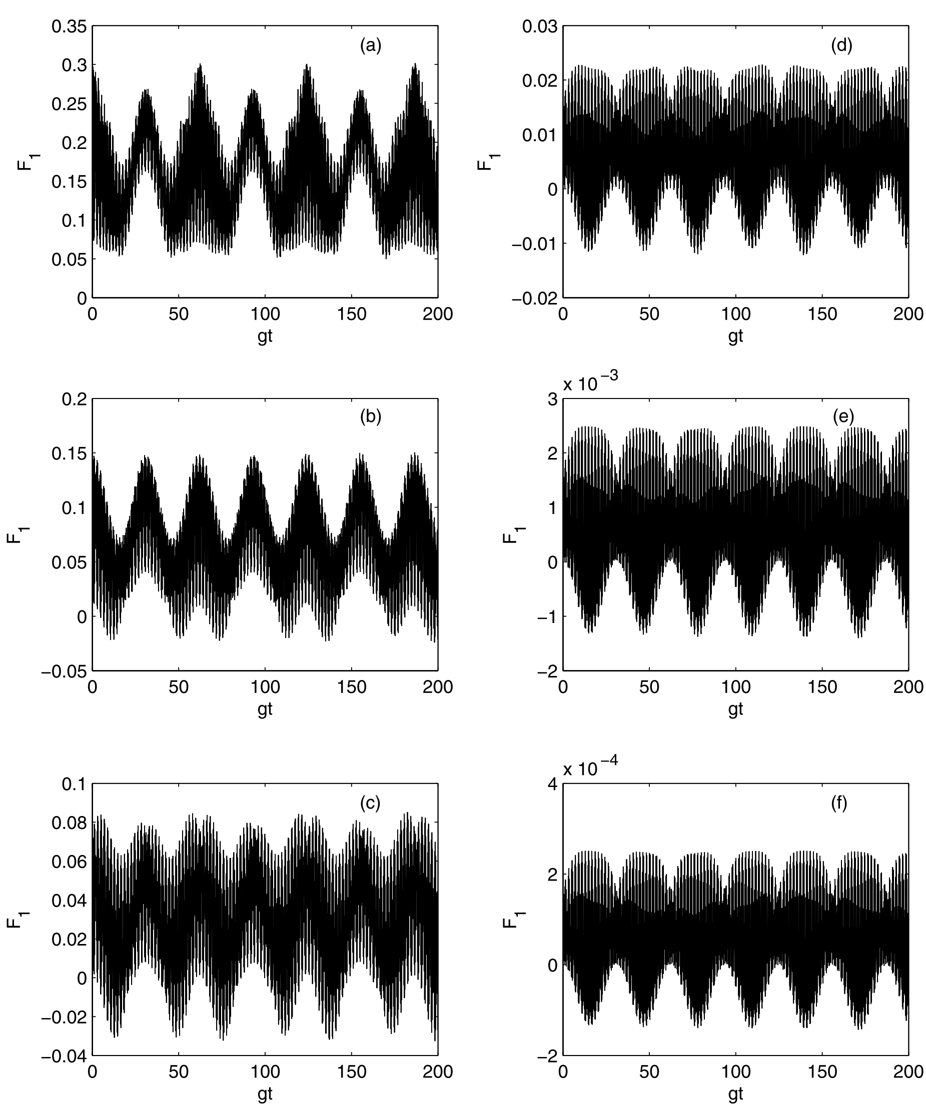

Fig. 1 Time evolution of F_1 for the maximum photon number $M = 100$, the characteristic probability (**a**) $p = 0.98$; (**b**) $p = 0.99$; (**c**) $p = 0.995$; (**d**) $p = 0.999$; (**e**) $p = 0.9999$; (**f**) $p = 0.99999$

evolution of F_2 for the maximum photon number $M = 100$, the atoms initially in Bell state $|\phi_A(0)\rangle_+$.

Through a lot of numerical computation, it is found that spin squeezing for J_y is exhibited in the certain area $0.96 < p < 1$. But the fluctuation of J_x isn't squeezed for any p. From these pictures, we can find the degree of spin squeezing is rapidly close to zero when p is close to 1. Comparing Fig. 2 with Fig. 1, we can observe that the degree of the maximal squeezing is larger than in Fig. 1.

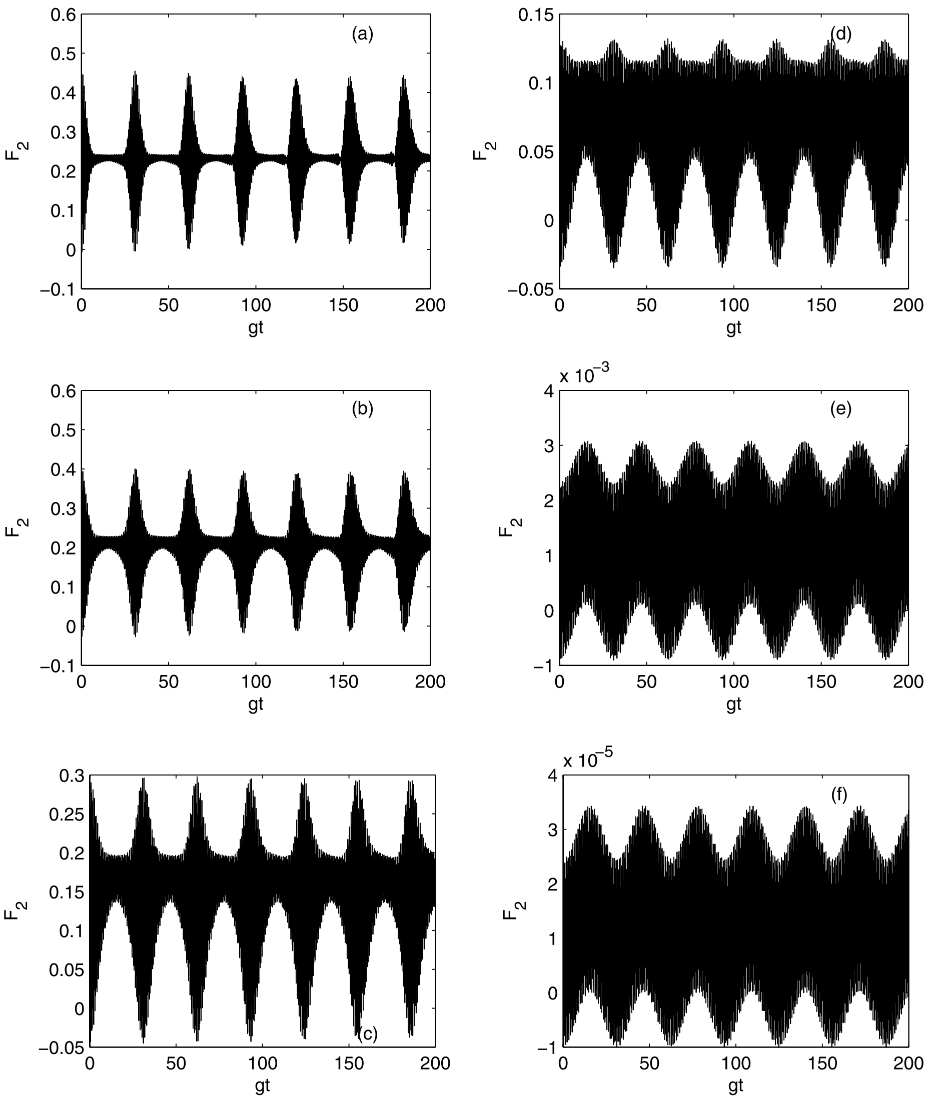

Fig. 2 Time evolution of F_2 for the maximum photon number $M = 100$, the characteristic probability (**a**) $p = 0.96$; (**b**) $p = 0.97$; (**c**) $p = 0.98$; (**d**) $p = 0.99$; (**e**) $p = 0.999$; (**f**) $p = 0.9999$

5 Conclusion

In this paper, we have investigated the spin squeezing of the two two-level atoms interacting with a binomial field. For the different initial conditions, the corresponding results are different. Firstly, F_1 (F_2) for the atoms initially in Bell state $|\varphi_A(0)\rangle_+$ and Bell state $|\varphi_A(0)\rangle_-$ always has the same numerical results and the same regulation is discovered in Bell state $|\phi_A(0)\rangle_+$ and Bell state $|\phi_A(0)\rangle_-$. Secondly, it is found that the fluctuation in the component J_y isn't squeezed for the atoms initially in Bell state $|\varphi_A(0)\rangle_+$ and the fluctuation in the component J_x isn't squeezed for the atoms initially in Bell state $|\phi_A(0)\rangle_+$. Finally, for the atoms initially in Bell state $|\varphi_A(0)\rangle_+$, spin squeezing for J_x is exhibited under the condition $0.98 < p < 1$. For the atoms initially in Bell state $|\phi_A(0)\rangle_+$, spin squeezing for J_y is exhibited under the condition $0.96 < p < 1$.

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